FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BPS-17 UNDER
THE FEDERAL GOVERNMENT, 2010

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: 3 HOURS	MAXIMUM MARKS:100

NOTE:	(i) Attempt **FIVE** questions in all by selecting at least **THREE** questions from **SECTION–A** and **TWO** questions from **SECTION–B**. All questions carry **EQUAL** marks. (ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) Let W be a subspace of a finite dimensional vector space V, then W is finite dimensional and dim (w) $\leq$ dim (v). Also if dim (w) = dim (V), then V = W. **(10)**

(b) Let V & W be vector space and let T : V → w be a linear if V is finite dimensional, then nullity (T) + rank (T) = dim v **(10)**

Q.2. (a) Show that there exist a homomorphism from S_n onto the multiplication group {-1,1} of 2 elements ($n \geq 1$). **(7)**

(b) If H is the only subgroup of a given finite order in a group G. Prove that H is normal in G. **(7)**

(c) Show that a field K has only two ideals (namely K & (o)). **(6)**

Q.3. (a) Find all possible jordan canonical forms for 3x3 matrix whose eiganvalues are -2,3,3 **(10)**

(b) Show that matrix $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ **(10)**

is diagonalizable with minimum calculation

Q.4. (a) Every group is isomorphic to permutation group **(7)**

(b) Show that for $n \geq 3$ $Z(s_n) = I$ **(6)**

(c) Let A, B be two ideal of a ring, then $\frac{A+B}{A} = \frac{B}{A \cap B}$. **(7)**

Q.5. (a) Verify Cayley – Hamilton theorem for the matrix **(7)**

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(b) Prove that ring A = $\underset{1}{Z}$, the set of all integers is a principal ideal ring. **(7)**

(c) Under what condition on the scalar, do the vectors (1,1,1), $(1, \xi, \xi^2)$, $(1, -\xi, \xi^2)$ form basis of c^3? **(6)**

SECTION – B

Q.6. (a) Show that T.N. = 0 for the helix **(10)**

$R(t) = (a\cos wt)\,\hat{z} + (a \sin wt)\,\hat{j} + (bt)\,\hat{k}$

(b) The vector equation of ellipse :$r(t) = (2 \cos t)\,\hat{i} + (3 \text{ Sint})\,\hat{j}$; $(0 \leq t \leq 2\Pi)$

Find the eurvature of ellipse at the end points of major & minor axes. **(10)**

Q.7. (a) Discuss & sketch the surface **(12)**

$x^2+4y^2=4x-4z^2$

(b) Show that an equation to the right circular cone with vertex at 0, axis oz & semi – vertical angle $\propto$ is $x^2+y^2=z^2 \tan^2 \propto$ **(8)**

Q.8. (a) Show that hyperboloids of one sheet and hyperbolic parabolas are ruled surface. **(6+6)**

(b) Find an equation of the plane which passes through the point (3,4,5) has an x – intercept equal to -5 and is perpendicular to the plane 2x+3y-z = 8. **(8)**

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Roll Number

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: 3 HOURS	**MAXIMUM MARKS:100**

NOTE:	(i) Attempt **FIVE** questions in all by selecting at least **THREE** questions from **SECTION–A** and **TWO** questions from **SECTION–B**. All questions carry **EQUAL** marks. (ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) If f is continuous on [a,b] and if $\propto$ is of bounded variation on [a,b], then $f \in R(\propto)$ on [a, b] i.e. f is Riemann – integrable with respect to $\propto$ on [a,b] **(10)**

(b) Let $\sum a_n$ be an absolutely convergent series having sum S. then every rearrangement of $\sum a_n$ also converges absolutely & has sum S. **(10)**

Q.2. (a) For what +ve value of P, $\int_0^1 \frac{dn}{(1-x)^p}$ is convergent? **(10)**

(b) Evaluate $\int_1^5 \frac{dx}{\sqrt[3]{x-2}}$ **(10)**

Q.3. (a) Find the vertical and horizontal asymptotes of the graph of function:
$f(x) = (2x+3)\sqrt{x^2-2x+3}$ **(10)**

(b) Let (i) $y = f(x) = \frac{(x+2)(x-1)}{(x-3)^2}$

(ii) $y=f(x) = \frac{(x-1)}{(x+3)(x-2)}$ **(10)**

Examine what happens to y when $x \to -\infty$ & $x \to +\infty$

Q.4. (a) Find a power series about 0 that represent $\frac{x}{1-x^3}$ **(6)**

(b) Let $\sum s_n$ be any series, Justify. **(5+5+4)**

(i) if $\lim_{n\to\infty} \left|\frac{Sn+1}{Sn}\right| = r < 1$, then $\sum s_n$ is absolutely convergent.

(ii) if $\lim_{n\to\infty} \left|\frac{Sn+1}{Sn}\right| = r$ and ($r > 1$ or $r=\infty$), then S_n diverges.

(iii) if $\lim_{n\to\infty} \left|\frac{Sn+1}{Sn}\right| =1$, then we can draw no conclusion about the convergence or divergence.

Q.5. (a) Show that $\int_{0}^{\Pi 12} Sin^{2m-1}\theta \cos^{2n-1} \theta d\theta = \frac{\Gamma(m)\,\Gamma(n)}{2\Gamma(m+n)}; m,n > 0$ **(10)**

(b) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}; m,n, > 0$ **(10)**

Q.6. (a) Let A be a sequentially compact subset of a matrix space X. Prove that A is totally bounded. **(10)**

(b) Let A be compact subset of a metric space (X,d) and let B be a closed subset of X such that $A \cap B = \Phi$ show that d(A,B)>0 **(10)**

SECTION – B

Q.7. (a) Show that if tanZ is expanded into Laurent series about $Z = \frac{\Pi}{2}$, then **(10)**

(i) Principal is $\frac{-1}{z - \Pi/2}$

(ii) Series converges for $0 <| Z - \frac{\Pi}{2} |< \frac{\Pi}{2}$

(b) Evaluate $\frac{1}{2\Pi i} \oint_{C} \frac{e^{zt}}{z^2\left(z^2 + 2z + 2\right)} dz$ around the circle with equation |z|=3. **(10)**

Q.8. (a) Expand f(x) = x^2 ; $0<x<2\Pi$ in a Fourier series if period is 2Π. **(10)**

(b) Show that $\int_{0}^{\infty} \frac{Cosxdx}{x^2+1} = \frac{\Pi}{a} e^{-x}; x \geq 0$ **(10)**

Q.9. (a) Let f(z) be analytic inside and on the simple close curve except at a pole of order m inside C. Prove that the residue of f(Z) at a is given by $a_{-1} = \lim_{Z \to a} \frac{1}{(m-1)!} \frac{{}^{m-1}d}{dz^{m-1}} \{(z-a)^m f(z)\}$ **(10)**

(b) If f(z) s analytic inside a circle C with center at a, then for all Z inside C.

$f(z) = f(a) + f^{/}(a)(z-a) + f^{//}\frac{(a)}{2!}(z-a)^2 + f^{///}\frac{(a)}{3!}(z-a)^3 + ...$ **(10)**

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