FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2012

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE:
(i) Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B**. **ALL** questions carry **EQUAL** marks.
(iii) Extra attempt of any question or any part of the attempted question will not be considered.
(iv) **Use of Scientific Calculator is allowed.**

SECTION-A

Q. 1. (a) Let H be a normal subgroup and K a subgroup of a group G. Prove that HK is a subgroup of G and $H \cap K$ is normal in K and $\dfrac{HK}{H} \cong \dfrac{K}{H \cap K}$. **(12)**

(b) Show that number of elements in a Conjugacy class Ca of an element 'a' in a group G is equal to the index of its normaliser. **(8)**

Q. 2. (a) Prove that if G is an Abelian group, then for all $a, b \in G$ and integers n, $(ab)^n = a^n b^n$. **(6)**

(b) Show that subgroup of Index 2 in a group G is normal. **(7)**

(c) If H is a subgroup of a group G, let $N(H) = \{a \in G \mid aHa^{-1} = H\}$ Prove that $N(H)$ is a subgroup of G and contains H. **(7)**

Q. 3. (a) Show that set C of complex numbers is a field. **(6)**

(b) Prove that a finite integral domain is a field. **(6)**

(c) Show that $Z_6 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$ is a ring under addition mod 6 and multiplication mod 6 but not a field. Find the divisors of Zero in Z_6. **(8)**

Q. 4. (a) Let F be a field of real numbers, show that the set V of real valued continuous functions on the closed interval $[0,1]$ is a vector space over F and the subset Y of V containing all functions whose nth derivatives exist, forms a subspace of V. **(10)**

(b) Prove that any finite dimensional vector space is isomorphic to F^n. **(10)**

Q. 5. (a) State and prove Cayley-Hamilton theorem. **(10)**

(b) Use Cramer's rule to solve the following system of linear equations: **(10)**

$$x + y + z + w = 1$$
$$x + 2y + 3z + 4w = 0$$
$$x + y + 4z + 5w = 1$$
$$x + y + 5z + 6w = 0$$

SECTION-B

Q. 6. (a) Prove that an equation of normal to the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ can be written in the form: **(10)**

$$y\,Cos\theta - x\,Sin\theta = a\,Cos2\theta$$

Hence show that the evolute of the curve is

$$(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$$

(b) If and are radii of curvature at the extremities of any chord of the Cardioid $r = a(1 + Cos\theta)$ which passes through the pole, then prove that $= \dfrac{16a^2}{9}$. **(10)**

PURE MATHEMATICS, PAPER-I

Q. 7. **(a)** Find an equation of the normal at any point of the curve with parametric equations: **(10)**
$x = a(Cos\,t + t\,Sin\,t), \quad y = a(Sin\,t - t\,Cos\,t)$.
Hence deduce that an equation of its evolute is $x^2 + y^2 = a^2$.

(b) Find equations of the planes bisecting the angle between the planes $3x + 2y - 6z + 1 = 0$ and $2x + y + 2z - 5 = 0$. **(10)**

Q. 8. **(a)** Define a surface of revolution. Write equation of a right elliptic-cone with vertex at origin. **(6)**

(b) Identify and sketch the surface defined by **(6)**
$x^2 + y^2 = 2z - z^2$.

(c) If *y=f(x)* has continuous derivative on $[a,b]$ and S denotes the length of the arc of *y=f(x)* between the lines *x=a* and *x=b*, prove that **(8)**

$$S = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,.$$

Find the length of the parabolas $y^2 4ax$

(i) From vertex to an extremity of the latus rectum.

(ii) Cut off by the latus rectum.

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Roll Number

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Candidate must write **Q. No.** in the **Answer Book** in accordance with **Q. No.** in the **Q. Paper.**
(ii) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B**. **ALL** questions carry **EQUAL** marks.
(iii) Extra attempt of any question or any part of the attempted question will not be considered.
(iv) Use of Scientific Calculator is allowed.

SECTION-A

Q. 1. (a) State and prove Taylor's theorem with Cauchy's form of remainder. **(8)**

(b) Evaluate (i) $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$ (ii) $\int e^{ax}\sin(bx+c)dx$ **(6)**

(c) Show that $\int_0^{\pi/2}\sin^p x\cos^q x\,dx = \dfrac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q}{2}+1\right)}$ **(6)**

Q. 2. (a) Sketch the graph of the curve $r^2 = a\,Sin 2\theta, a > 0$. Also write pedal equation for this curve. **(8)**

(b) Show that the parabola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ has asymptotes $y=\frac{b}{a}x$ and $y=-\frac{b}{a}x$ **(6)**

(c) Define extrema (local and global) of a function of two variables. Find three positive numbers whose sum is 48 and whose product is as large as possible. **(6)**

Q. 3. (a) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, a, $b, c > 0.$ **(8)**

(b) Evaluate $\int_0^{\pi/2}\ell n(Sin\,x)dx$ **(6)**

(c) Determine the values of *x* for which the power series $\sum_{n=2}^{\infty}\frac{x^n}{\ell n\; n}$ converges absolutely, converges conditionally and diverges. **(6)**

Q. 4. (a) Define a metric on a non-empty set *X*. If *d* is a metric on *X*, show that if $d'(x,y)=\dfrac{d(x,y)}{1+d(x,y)}$ then d' is also a metric on *X*. Also write open and closed balls (spheres) in the discrete metric space (*X*, do) with radius 1 and 1.1 centered at some $x \in X$. **(5+3+2 =10)**

(b) Define limit point of a subset *A* of a metric space *X*. Show that an open sphere containing a limit point *x* of *A* contains infinitely many points of *A* other than *x*. **(10)**

PURE MATHEMATICS, PAPER-II

Q. 5. **(a)** Show that R^n is a complete metric space under the metric defined by **(7)**

$d(x,y)=\sqrt{\sum(\xi_i-\eta_i)^2},\ x,y\in R^n$

Where $x=(\xi_1,\xi_2,.......,\xi_n)$ and $y=(\eta_1,\eta_2,.......,\eta_n)$

(b) Show that a function $f:(X.d)\rightarrow(Y,d')$ is continuous if and only if for an open subset V of Y, $f^{-1}(V)$ is an open subset of X. **(7)**

(c) Find the radius of convergence and interval of convergence of the power series: **(6)**

$$\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(x+1)^{2n}}{(n+1)^2\ 5^n}$$

SECTION-B

Q. 6. **(a)** If C is a continuous curve and f(z) is defined on each point of C, then prove that **(10)**

$$\left|\int_C f(z)dz\right|\le ML$$

Where $M=max|f\ z|$ and L is length of curve C.

(b) Suppose $f(z)=U(x,y)+iV\ \ (x,y)$ is differentiable at a point $z=x+iy$, then at z the first order partial derivatives of U an V exist and satisfy Cauchy-Reiman equations: $\frac{\partial U}{\partial x}=\frac{\partial\ V}{\partial y},\frac{\partial U}{\partial y}=-\frac{\partial V}{\partial x}$. **(10)**

Verify Cauchy-Reiman equations for the function $f(z)=e^{-x}\cos y-i\,e^{-x}\sin y$.

Q. 7. **(a)** Define singularity of a function $f(z)$. Investigate for the pole, singularities and zeros, the function $f(z)=z^2$ **(6)**

(b) Let D be simply connected domain and $f(z)$ be analytic in D. Let $f'(z)$ exist and is continuous at each point of D then prove that $\int_c f(z)dz=0$, where C is any closed Contuor in D. **(6)**

(c) State De Moivre's theorem and hence prove that **(8)**

(i) $Cos\,5\theta=16Cos^3\theta-20Cos^2\theta+5Cos\theta$

(ii) $Sin^n\theta=(-1)^{\frac{n-1}{2}}\frac{1}{2^{n-1}}\left[Sin\,n\theta-Sin(n-2)\theta+\frac{n(n-1)}{2}Sin(n-4)\theta-.........\right]$

Q. 8. **(a)** Solve the equation x^{12}-1=0 and find which of its roots satisfy the equation x^4+x^2+1=0. **(6)**

(b) Show that multiplication of a vector z by $e^{i\alpha}$ where α is a real number, rotates the vector z counter clockwise through an angle of measure α. **(6)**

(c) Sum the series **(8)**

$$nSin\theta+\frac{n(n+1)}{2!}Sin2\theta+\frac{n(n+1)(n+2)}{3!}Sin3\theta+.........$$

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