FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2014

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE:(i) Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B. ALL** questions carry **EQUAL** marks.

(ii) Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper.**

(iii) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.

(iv) Extra attempt of any question or any part of the attempted question will not be considered.

(v) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** If G is a group in which $(a \cdot b)^i = a^i \cdot b^i$ for three consecutive integers i for all $a, b \in G$, show that G is abelian. **(10)**

(b) The center Z of a group G is defined by $Z = \{z \in G / zx = xz \; all \; x \in G\}$. Prove that Z is a subgroup of G. **(10)**

Q. No. 2. **(a)** If $f : G \to G'$ be a homomorphism. Prove that $Ker\ f$ is a normal subgroup of G. **(10)**

(b) Prove that any group of order 15 is cyclic. **(10)**

Q. No. 3. **(a)** If in a ring R with unity, $(xy)^2 = x^2 y^2$ for all $x, y \in R$, then show that R is commutative. **(10)**

(b) Prove that the set $Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$ forms a commutative ring with unit element under addition and multiplication module 7. **(10)**

Q. No. 4. **(a)** Prove that a non empty subset W of a vector space $V(F)$ is a subspace of V if and only if $\sim x + \quad y \in W$ for $\sim, \quad \in F$, $x, y \in W$. **(10)**

(b) Show that the vectors $v_1 = (1, -1, -4, 0)$, $v_2 = (1, 1, 2, 4)$, $v_3 = (2, -1, -5, 2)$, $v_4 = (2, 1, 1, 6)$ are linearly dependent in $\mathbb{R}^4(\mathbb{R})$. **(10)**

Q. No. 5. **(a)** A company produces three products, each of which must be processed through three different departments. Given table summarizes the hours required per unit of each product in each department. In addition, the weekly capacities are stated for each department in terms of work-hours available. What is desired is to determine whether there are any combinations of the three products which would exhaust the weekly capacities of the three departments. **(10)**

Department	Product 1	Product 2	Product 3	Hours Available per Week
A	2	3.5	3	1,200
B	3	2.5	2	1,150
C	4	3	2	1,400

(b) Show that $\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$ **(10)**

SECTION-B

Q. No. 6. **(a)** Find the equation of the straight line joining two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angles are given. Hence find equations of the tangent and normal at any point ¯ on the ellipse. **(10)**

(b) Find the angle of intersection of the cardioids $r=a(1+\cos$ ¯ $)$ and $r=b(1-\cos$ ¯ $)$. **(10)**

Q. No. 7. **(a)** Find the equation of the line L through the point $(5, \frac{7}{2}, 5)$ and intersecting at right angles the line M with parametric equ ations $x=4+3t, y=1+t, z=-3t$. **(10)**

(b) Find the equation of the tangent plane at any point $P(x_1, y_1, z_1)$ of the elliptic paraboloid $z=x^2+4y^2$. **(10)**

Q. No. 8. **(a)** Find the volume of the solid obtained by revolving the area enclosed by one arc of the cycloid $x=a($ ¯ $+\sin$ ¯ $)$, $y=a(1+\cos$ ¯ $)$ about $x-axis$. **(10)**

(b) Discuss the surface and make a sketch, $x^2-y^2+z^2=1$. **(10)**

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Roll Number

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:(i) Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper.**
(ii) Attempt FIVE questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B. ALL** questions carry **EQUAL** marks.
(iii) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(iv) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed.**

SECTION-A

Q. No. 1. **(a)** Prove that if n is a positive integer which is not a perfect square, then $\sqrt{n}$ is an irrational number. **(10)**

(b) Show that every non-empty set of real numbers which has a lower bound has the infimum. **(10)**

Q. No. 2. **(a)** For what value of a, m, and b does the function **(10)**

$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$

Satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$? **(10)**

(b) For what value of a is

$$f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2ax & x \ge 3 \end{cases}$$

Continuous at every x?

Q. No. 3. **(a)** Find the area of the surface generated by revolving $r = 2aSin$¯ about the polar axis. **(6)**

(b) Find the area enclosed by the graph of the cardioid $r = a(1 - Sin$¯$)$. **(7)**

(c) Evaluate the integral $\int_1^{10} \frac{dx}{(x-2)^{2/3}}$ **(7)**

Q. No. 4. **(a)** Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. **(6)**

(b) For what value of x does the series converges absolutely, converges conditionally and diverges? **(7)**

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2 + 3}}$$

(c) Let $f(x,y)=\begin{cases} \dfrac{x^3}{x^3+y^3} & if\ (x,y)\neq(0,0) \\ 0 & if\ (x,y)=(0,0) \end{cases}$ **(7)**

Show that f is not continuous at the origin.

Q. No. 5. **(a)** Let x be a non-empty set and define **(10)**
$d : X_x X \rightarrow R$ by

$$d(a,b)=\begin{cases} 1 & if\ a\neq b \\ 0 & if\ a=b \end{cases}$$

Show that d is a metric on X
Also describe open and closed balls in this metric space.

(b) Prove that a function f from a metric space (x, d) into a metric space (Y, d^1) is continuous if and only if $f^{-1}(A)$ is a closed subset of X for every closed subject A of Y. **(10)**

SECTION-B

Q. No. 6. **(a)** Using De Moivre's Theorem evaluate **(10)**

$$\left(\frac{1+i}{\sqrt{3}+i}\right)^6$$

(b) Find real constants a, b, c and d so that the given function is analytic **(10)**

$$f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$$

Q. No. 7. **(a)** Evaluate $\oint_c \dfrac{dz}{z^2+1}$, where c is the circle $|z| = 4$. **(10)**

(b) Expand $(z)=\dfrac{1}{z(z-1)}$. in a Laurent series valid for $1<|z-2|<2$. **(10)**

Q. No. 8. **(a)** Find the Fourier transform of $f(z)=e^{-|x|}$. **(10)**

(b) Evaluate $\oint_c \dfrac{1}{(z-1)^2(z-3)}dz$, where the contour C is the rectangle defined by $x= 0, x\ 4, y= -1, y=1$ **(10)**

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