FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2017
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. (a) Let H, K be subgroups of a group G. Prove that HK is a subgroup of G if and only if $HK=KH$. (10)

(b) If N, M are normal subgroups of a group G, prove that (10) **(20)**

$$NM/M \cong N/N\cap M.$$

Q. 2. (a) If R is a commutative ring with unit element and M is an ideal of R then show that M is a maximal ideal of R if and only if R/M is a field. (10)

(b) If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then show that we can find elements a and b in F such that (10) **(20)**

$$1 + \alpha a^2 + \beta b^2 = 0.$$

Q. 3. (a) Let V be a finite-dimensional vector space over a field F and W be a subspace of V. Then show that W is finite-dimensional, (10)

$$\dim W \leq \dim V \text{ and } \dim V/W = \dim V - \dim W.$$

(b) Suppose V is a finite-dimensional vector space over a field F. Prove that a linear transformation $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0. (10) **(20)**

SECTION-B

Q. 4. (a) Use the Mean-Value Theorem to show that if f is differentiable on an interval I, and if $|f'(x)| \leq M$ for all values of x in I, then (10)

$$|f(x) - f(y)| \leq M|x - y|$$

for all values of x and y in I. Use this result to show further that

$$|\sin x - \sin y| \leq |x - y|.$$

(b) Prove that if $x = x(t)$ and $y = y(t)$ are differentiable at t, and if $z = f(x, y)$ is differentiable at the point $(x, y) = \big(x(t), y(t)\big)$, then $z = f\big(x(t), y(t)\big)$ is differentiable at t and (10) **(20)**

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y).

Q. 5. (a) Evaluate the double integral (10)

$$\iint_R (3x - 2y)\, dx\, dy$$

(b) Where R is a region enclosed by the circle $x^2 + y^2 = 1$. (10) **(20)**

Find the area of the region enclosed by the curves

$$y = \sin x, \quad y = \cos x, x = 0, \quad x = 2\pi.$$

Q. 6. **(a)** Find an equation of the ellipse traced by a point that moves so that the sum of its distance to (4,1) and (4,5) is 12. (10)

(b) Show that if a, b and c are nonzero, then the plane whose intercepts with the coordinate axes are $x = a$, $y = b$, and $z = c$ is given by the equation. (10) **(20)**

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

SECTION-C

Q. 7. **(a)** Prove that a necessary and sufficient condition that (10)

$$w = f(z) = u(x, y) + iv(x, y)$$

be analytic in a region R is that the Cauchy-Riemann equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{and} \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

are satisfied in R where it is supposed that these partial derivatives are continuous in R.

(b) Show that the function $f(z) = \bar{z}$ is not analytic anywhere in the complex plane Z. (10) **(20)**

Q. 8. **(a)** Let $f(z)$ be analytic inside and on the boundary C of a simply-connected region R. Prove that (10)

$$f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz.$$

(b) Show that (10) **(20)**

$$\int_0^{2\pi} \frac{d\theta}{(5 - 3\sin\theta)^2} = \frac{5\pi}{32}.$$

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